# Influence of Water on Esterification Reaction between Terephthalic Acid and Ethylene Glycol 

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## Synopsis

Examples of simulation in the first reactor for the direct continuous esterification were shown to predict the effects of water added into the reactor ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) on the reactions by a computer. From the simulation results, the followings were confirmed: Although the higher $\delta_{\mathrm{H}_{2} \mathrm{O}}$ gives the worse results, i.e., the main reaction rates become lower, it brings about the better results, i.e., the side reactions of diethylene glycol (DEG) formation is suppressed; the clearing point where solid (undissolved) terephthalic acid begins not to exist in the reaction mixtures, $\beta=1$, is the point of $A V$ $=0.6 \mathrm{eq} / \mathrm{kg}$, which coincides with the point obtained from the experimental observation, independently of reaction conditions.

## INTRODUCTION

When manufacturing poly (ethylene terephthalate) (PET) continuously, numerous conditions, such as the charge rates of raw and auxiliary materials, temperatures, pressures, etc., are involved and the product, PET, quality or production rate are more or less dependent on each of these conditions.

The simulation model of continuous esterification reaction proposed by Yamada et al. ${ }^{1}$ has been utilized to conduct an integrated analysis of the effects of each individual condition and ascertain the optimum manufacturing conditions in the production plant. In other words, it is a computer software to estimate the combination of conditions such as rates of raw and auxiliary materials, concentrations, reaction temperatures, reaction pressures, etc. in the esterification reactors for continuous manufacturing of PET through simulating actual situations by a computer. The present model gives useful information on the continuous esterification reaction shown as follows.

## Input Data (Operational Conditions)

1. Reaction temperature: $t$;
2. Reaction pressure: $P$;
3. Throughput based on PET: $F_{s}$;
4. Total weight in the reactor: $W$ (or mean residence time: $\tau$ );
5. Molar ratio of ethylene glycol (EG) to terephthalic acid (TPA), which fed to the first reactor: $E / T$;
6. Number of reactors: $n$;
7. Flow rate of EG added anew to the reactor per $1 \mathrm{~kg} / \mathrm{h}$ of throughput based on PET polymer: $\delta_{\mathrm{EG}}$;
8. Flow rate of water added anew to the reactor per $1 \mathrm{~kg} / \mathrm{h}$ of throughput based on PET polymer: $\delta_{\mathrm{H}_{2} \mathrm{O}}$.

## Output Information

1. Properties of dried oligomer: $A V, O H V, e, d, \phi, P_{n}, M_{n}, E_{s}, S V$
$A V=$ concentration of carboxyl end groups,
$O H V=$ concentration of hydroxyl end groups,
$e=$ molar ratio of diethylene glycol (DEG) content to bound EG,
$d=$ percentage of DEG content based on TPA,
$\phi=$ ratio of hydroxyl end groups to total end groups,
$P_{n}=$ number average degree of polymerization,
$M_{n}=$ number average molecular weight,
$E_{s}=$ esterification degree,
$S V=$ saponification value,
2. Concentrations of each component in the liquid phase: $\mathrm{C}_{1}-\mathrm{C}_{8}$;
3. Free EG and water contents in the reaction mixtures: $w_{\mathrm{EG}}, w_{\mathrm{H}_{2} \mathrm{O}}$;
4. Vapor flow rates from the reactor: $Q, Q_{\mathrm{EG}}, Q_{\mathrm{H}_{2} \mathrm{O}}$;
5. Compositions of vapor phase: $w_{\mathrm{EG}}^{d}, w_{\mathrm{H}_{2} \mathrm{O}}^{d}$;
6. Flow rate of the reaction mixtures fed to the next reactor: $F_{P}$;
7. Weight fraction of the liquid phase in the reaction mixtures: $\beta$;
8. Melting point of oligomer: $T_{\mathrm{m}}$.

In manufacturing PET, EG not consumed by the reactions is usually reused as raw material from the economical viewpoint, as proposed in the patent by Imamura et al. ${ }^{2}$ In this process it is important to understand the influence of water in recovered EG on the esterification reaction. The author estimates the influence of water on the esterification reaction in the first reactor by the computer simulation based upon the model proposed by Yamada et al., ${ }^{1}$ the reaction rate constants determined by Yamada et al. ${ }^{3}$ and Yamada, ${ }^{4}$ and the relations between the melting point and properties of oligomer reported by Yamada et al. ${ }^{5,6}$

## REACTION SCHEME FOR DIRECT ESTERIFICATION

Yamada et al. ${ }^{1}$ assumed that various reactions occurring in an esterification reactor are as given below.

1. Esterification reactions

2. Polycondensation reactions

3. Side reactions leading to the formation of diethylene glycol (DEG) in the oligomer

$C_{5}$

$2 \mathrm{HOCH}_{2} \mathrm{CH}_{2} \mathrm{OH} \xrightarrow{\mathrm{k}_{0}} \mathrm{HOCH}_{2} \mathrm{CH}_{2} \mathrm{OCH}_{2} \mathrm{CH}_{2} \mathrm{OH}+\mathrm{H}_{2} \mathrm{O}$


 $C_{7} \quad C_{5}$
where $C_{i}$ is the concentration of component $i$ in the liquid phase and $k_{1}$ to $k_{9}$ are the reaction rate constants, and there are the relations $k_{8}=2 k_{7}$ and $k_{9}=4 k_{7}$ (see Appendix A).

## REACTION RATE EQUATIONS

The reaction rate equations of $C_{1}$ to $C_{8}$ are given as follows.

$$
\begin{array}{r}
\left(d C_{1} / d \theta\right)=-k_{1} C_{1} C_{2}+k_{2} C_{3} C_{5}-k_{3} C_{1} C_{3}+k_{4} C_{4} C_{5}-k_{1} C_{1} C_{8}+k_{2} C_{5} C_{7} \\
\left(d C_{2} / d \theta\right)=-k_{1} C_{1} C_{2}+k_{2} C_{3} C_{5}+k_{5} C_{3}^{2}-k_{6} C_{2} C_{4}-k_{8} C_{2} C_{3}-2 k_{9} C_{2}^{2} \\
\left(d C_{3} / d \theta\right)=k_{1} C_{1} C_{2}-k_{2} C_{3} C_{5}-k_{3} C_{1} C_{3}+k_{4} C_{4} C_{5} \\
-2 k_{5} C_{3}^{2}+2 k_{6} C_{2} C_{4}-2 k_{7} C_{3}^{2}-k_{8} C_{2} C_{3} \\
\left(d C_{4} / d \theta\right)=k_{3} C_{1} C_{3}-k_{4} C_{4} C_{5}+k_{5} C_{3}^{2}-k_{6} C_{2} C_{4} \\
\left(d C_{5} / d \theta\right)=k_{1} C_{1} C_{2}-k_{2} C_{3} C_{5}+k_{3} C_{1} C_{3}-k_{4} C_{4} C_{5} \\
+k_{7} C_{3}^{2}+k_{8} C_{2} C_{3}+k_{9} C_{2}^{2}+k_{1} C_{1} C_{8}-k_{2} C_{5} C_{7} \\
\left(d C_{6} / d \theta\right)=k_{7} C_{3}^{2} \\
\left(d C_{7} / d \theta\right)=k_{8} C_{2} C_{3}+k_{1} C_{1} C_{8}-k_{2} C_{5} C_{7} \\
\left(d C_{8} / d \theta\right)=k_{9} C_{2}^{2}-k_{1} C_{1} C_{8}+k_{2} C_{5} C_{7} \tag{15}
\end{array}
$$

where $k_{8}=2 k_{7}$ and $k_{9}=4 k_{7}$, and $\theta$ denotes the reaction time.

## MATHEMATICAL MODEL FOR CONTINUOUS REACTORS

## Assumptions for Modeling

Assumptions for modeling of direct esterification are given as follows.

1. Reactions occur only in the liquid phase. TPA is partly dissolved in the reaction mixtures and this dissolved TPA participates in the reaction.
2. The rate of dissolution of TPA is very fast. The rate-determination step, therefore, is not the dissolution of TPA, but the reaction of TPA.
3. Only undissolved TPA forms the solid component of the heterogeneous system.
4. All acidic functions entail carboxyl end groups.
5. The concentration of DEG is given by the sum of $C_{6}, C_{7}$, and $C_{8}$.
6. The pressure in the reactor is given only by the sum of partial pressures of EG and water existing in it. The partial pressure of DEG is lower than those of EG and water.
7. All the nonvolatile components in reaction mixtures are defined as oligomer.
8. Reactivity of functional group does not depend on the polymer chain length as it can be assumed that the degree of polymerization is not very high in the esterification step.
9. In the formation of DEG the reactivity of each hydroxyl end group will be assumed to be the same. That is, $k_{8}=2 k_{7}$ and $k_{9}=4 k_{7}$.

## Liquid Weight Fraction in Reaction Mixtures ( $\beta$ )

Since the esterification of TPA with EG is considered to occur only in the liquid phase, unaffected by any undissolved TPA, the weight fraction of this phase has been selected a principal parameter in the development of our simulation model.

The weight fraction of liquid phase, $\beta$, in the reaction mixtures is related to the concentration of carboxyl end groups in the system by eq. (16), where $A V_{a}$ represents the total concentration of carboxyl end groups in the reaction reaction mixtures and $A V$ represents their concentration based upon the weight of the mixtures excluding that of both EG and water.

$$
\begin{equation*}
A V_{a}=A V \cdot\left(1-\beta w_{\mathrm{EG}}-\beta w_{\mathrm{H}_{2} \mathrm{O}}\right) \tag{16}
\end{equation*}
$$

When there is no undissolved TPA in reaction mixtures ( $\beta=1$ ), eq. (16) reduces to eq. (17)

$$
\begin{equation*}
\left(A V_{a}\right)_{\beta=1}=\zeta=A V \cdot\left\{1-\left(w_{\mathrm{EG}}+w_{\mathrm{H}_{2} \mathrm{O}}\right)\right\}=A V \cdot w_{\mathrm{OLG}} \tag{17}
\end{equation*}
$$

where $w_{\mathrm{EG}}=$ weight fraction of EG in the liquid phase of reaction mixtures, $w_{\mathrm{H}_{2} \mathrm{O}}=$ weight fraction of water in the liquid phase of reaction mixtures, and $w_{\text {OLG }}=$ weight fraction of oligomer in the liquid phase of reaction mixtures.
$w_{\mathrm{EG}}, w_{\mathrm{H}_{2} \mathrm{O}}$ and $w_{\mathrm{OLG}}$ satisfy the following equation.

$$
\begin{equation*}
w_{\mathrm{EG}}+w_{\mathrm{H}_{2} \mathrm{O}}+w_{\mathrm{OLG}}=1 \tag{18}
\end{equation*}
$$

When the mean solubility of TPA in the liquid phase of reaction mixtures is represented by $\alpha$, the following relationships results.

1. $\zeta>\alpha$ (when insoluble TPA remains in reaction mixtures)

$$
\begin{equation*}
\alpha \beta=A V_{a}-\eta_{\mathrm{TPA}} \cdot(1-\beta) \tag{19}
\end{equation*}
$$

Substitution of eq. (16) into eq. (19) gives

$$
\begin{equation*}
\beta=\frac{A V-\eta_{\mathrm{TPA}}}{\alpha+A V \cdot\left(w_{\mathrm{EG}}+w_{\mathrm{H}_{2} \mathrm{O}}\right)-\eta_{\mathrm{TPA}}} \tag{20}
\end{equation*}
$$

where $\eta_{\text {TPA }}=$ equivalents of carboxyl end groups per 1 kg pure TPA (i.e., $\eta_{\text {TPA }}$ $=12.039-\mathrm{COOH}$ equiv $/ \mathrm{kg}$ TPA ).

The concentration of carboxyl end groups in the reaction mixtures, $C_{1}$, is given by $\alpha$.
2. $\zeta \leqq \alpha$ (no undissolved TPA in reaction mixtures)

$$
\begin{equation*}
\beta=1 \quad \text { and } \quad C_{1}=\zeta \tag{21}
\end{equation*}
$$

where $\alpha$ is given as equivalent moles of carboxyl end groups of dissolved TPA in 1 kg liquid phase of reaction mixtures.

## Mean Solubility of TPA ( $\alpha$ )

The temperature dependences of the solubility of TPA in EG, $\alpha_{E G}$, and that in BHET, $\alpha_{\text {BHET }}$, are given as follows ${ }^{1}$

$$
\begin{align*}
\alpha_{\mathrm{EG}} & =18124 \cdot \exp (-9692 / R T)  \tag{22}\\
\alpha_{\mathrm{BHET}} & =758 \cdot \exp (-7612 / R T) \tag{23}
\end{align*}
$$

where

$$
\begin{aligned}
\alpha_{\mathrm{EG}} & =\text { solubility of TPA in EG; } \\
\alpha_{\mathrm{BHET}} & =\text { solubility of TPA in BHET; } \\
R & =\text { gas constant, } R=1.987 \mathrm{cal} / \mathrm{mol} \cdot{ }^{\circ} \mathrm{K} ; \\
T & =\text { absolute temperature, } T=t+273.15 .
\end{aligned}
$$

The mean solubility of TPA in reaction mixtures containing EG and water can be calculated by the following equation

$$
\begin{equation*}
\alpha=\alpha_{\mathrm{H}_{2} \mathrm{O}} \cdot w_{\mathrm{H}_{2} \mathrm{O}}+\alpha_{\mathrm{EG}} \cdot w_{\mathrm{EG}}+\alpha_{\mathrm{OLG}} \cdot w_{\mathrm{OLG}} \tag{24}
\end{equation*}
$$

where $\alpha_{\mathbf{H}_{2} \mathrm{O}}=$ solubility of TPA in water and $\alpha_{\mathrm{OLG}}=$ solubility of TPA in oligomer.

If the amount of dissolved TPA in the water component of reaction mixtures is negligible because of the very little amount of water in them and the solubility of TPA in oligomer, $\alpha_{\mathrm{OLG}}$, is equivalent to that in BHET, $\alpha_{\mathrm{BHET}}, \alpha$ is given by

$$
\begin{equation*}
\alpha \fallingdotseq \alpha_{\mathrm{EG}} \cdot w_{\mathrm{EG}}+\alpha_{\mathrm{BHET}} \cdot w_{\mathrm{OLG}} \tag{25}
\end{equation*}
$$

## Relations among Oligomer Properties

The relations among oligomer properties are given in the following equations

$$
\begin{align*}
\phi & =O H V /(A V+O H V)  \tag{26}\\
M_{n} & =2000 /(A V+O H V) \tag{27}
\end{align*}
$$

The chemical structure of oligomer with $P_{n}$ of polymerization degree can be expressed as follows

where atomic weights of carbon, hydrogen, and oxygen are $12.011,1.008$ and 15.999 , respectively.

The relation between $M_{n}$ and $P_{n}$ can be given by eq. (28)

$$
\begin{align*}
M_{n}=\left(P_{n}-1\right)(164.12+28.05) & /(1+e)+\left(P_{n}-1\right)(164.12+72.11) e / \\
(1+e)+ & 164.12+2(1-\phi) \times 1.008 \\
+ & 2 \phi\{45.05 /(1+e)+89.11 e /(1+e)\} \tag{28}
\end{align*}
$$

By rearrangement of eq. (28), $P_{n}$ can be expressed by eq. (29)

$$
\begin{align*}
& P_{n}=\left\{M_{n}(1+e)+26.03+70.09 e-\phi(88.10+176.20 e)\right\} / \\
& (192.17+236.23 e)  \tag{29}\\
& S V=2000 \cdot P_{n} / M_{n}  \tag{30}\\
& E_{s}=(S V-A V) / S V
\end{align*}
$$

As the sum of EG linkages and ether linkages (DEG) can be expressed as ( $P_{n}-1+2 \phi$ ), if the number of ether linkages is represented by $x, x$ is found by the following relation

$$
\begin{equation*}
(1+e): e=\left(P_{n}-1+2 \phi\right): x \text {, i.e., } x=\left(P_{a}-1+2 \phi\right) e /(1+e) \tag{32}
\end{equation*}
$$

As the number of TPA units is equal to $P_{n}, d$ is found by $100 \cdot x / P_{n}$.

$$
\begin{equation*}
d=100\left[\left(P_{n}+2 \phi-1\right) / P_{n}\right] \cdot[e /(1+e)] \tag{33}
\end{equation*}
$$

where $\phi=$ ratio of hydroxyl end groups to total end groups, $M_{n}=$ number average molecular weight, $P_{n}=$ number average degree of polymerization, $S V$ $=$ saponification value, $E_{s}=$ esterification degree, $e=$ molar ratio of DEG to bound EG, and $d=$ percentage of DEG content based on TPA.

The value of $e$ is measured by a gas chromatographic analysis after methanolysis of oligomer.

## PREDICTION OF MELTING POINTS

The following equation reported by Yamada et al. ${ }^{5,6}$ can give the prediction of melting points.

$$
\begin{align*}
1 / T_{m}= & 0.111049 \times 10^{-3} \cdot O H V^{2}-0.200468 \times 10^{-4} \cdot O H V^{3}-0.298614 \\
& \times 10^{-3} \cdot \exp (-O H V)-0.196193 \times 10^{-2} \cdot A V /\left(P_{n}+O H V\right) \\
& +0.605210 \times 10^{-3} \cdot A V^{2} /\left(P_{n}+O H V\right)^{2}-0.971548 \times 10^{-2} \cdot A V / \\
& \left(P_{n}+O H V\right)^{2}+0.980871 \times 10^{-3} \cdot O H V /\left(P_{n}+O H V\right)^{2}+0.142510 \\
& \times 10^{-3} \cdot \exp \left\{A V /\left(P_{n}+O H V\right)\right\}-0.655692 \times 10^{-4} \cdot \ln \left(P_{n}\right) \\
& +0.415790 \times 10^{-2} \tag{34}
\end{align*}
$$

where $T_{m}$ is the melting points of oligomer.

## MATERIAL BALANCE IN CONTINUOUS STIRRED TANK REACTOR (CSTR)

## Determination of Mole Fractions of EG and Water in the Vapor or Liquid Phases

If $q_{\mathrm{EG}}$ and $q_{\mathrm{H}_{2} \mathrm{O}}$ represent, respectively, the amount of EG consumed in reaction with TPA and water generated in the reaction, then for the case of a continuous stirred tank reactor (CSTR) the following equations may be written

$$
\begin{align*}
q_{\mathrm{EG}} & =\frac{62.07}{166.13} \cdot F_{\mathrm{TPA}} \cdot\left[\left(P_{\mathrm{nf}}+2 \phi_{f}-1\right) / P_{\mathrm{nf}}-\left(P_{\mathrm{ni}}+2 \phi_{i}-1\right) / P_{\mathrm{ni}}\right]  \tag{35}\\
q_{\mathrm{H}_{2} \mathrm{O}} & =\frac{2 \times 18.02}{166.13} \cdot F_{\mathrm{TPA}} \cdot\left[\left(P_{\mathrm{nf}}+\phi_{f}-1\right) / P_{\mathrm{nf}}-\left(P_{\mathrm{ni}}+\phi_{i}-1\right) / P_{\mathrm{ni}}\right] \tag{36}
\end{align*}
$$

where $F_{\text {TPA }}$ is the throughput based on TPA and subscripts $i$ and $f$ show the input and output, respectively. The derivation of eqs. (35) and (36) is given in Appendix B.

Material balances of EG and water in a CSTR are given as follows

$$
\begin{align*}
F_{\mathrm{EG}}^{i} & =Q_{\mathrm{EG}}+\beta F_{P} w_{\mathrm{EG}}+q_{\mathrm{EG}}  \tag{37}\\
F_{\mathrm{H}_{2} \mathrm{O}}^{i}+q_{\mathrm{H}_{2} \mathrm{O}} & =Q_{\mathrm{H}_{2} \mathrm{O}}+\beta F_{P} w_{\mathrm{H}_{2} \mathrm{O}} \tag{38}
\end{align*}
$$

Total material balance is

$$
\begin{equation*}
F^{i}=F_{P}+Q_{\mathrm{EG}}+Q_{\mathrm{H}_{2} \mathrm{O}} \tag{39}
\end{equation*}
$$

where
$F_{\mathrm{EG}}^{i}=$ input rate of EG,
$Q_{\mathrm{EG}}=$ vapor flow rate of EG ,
$F^{i}=$ total input rate,
$F_{\mathrm{H}_{2} \mathrm{O}}^{i}=$ input rate of water,
$Q_{\mathrm{H}_{2} \mathrm{O}}=$ vapor flow rate of water, and
$F_{P}=$ output rate of reaction mixtures.
Combining eqs. (37), (38), and (39) gives

$$
\begin{align*}
& Q_{\mathrm{EG}}=\left\{\left(F_{\mathrm{EG}}^{i}-q_{\mathrm{EG}}\right)\left(1-\beta w_{\mathrm{H}_{2} \mathrm{O}}\right)-\beta w_{\mathrm{EG}}\left(F^{i}-F_{\mathrm{H}_{2} \mathrm{O}}^{i}-q_{\mathrm{H}_{2} \mathrm{O}}\right)\right\} / \\
& \quad\left(1-\beta w_{\mathrm{H}_{2} \mathrm{O}}-\beta w_{\mathrm{EG}}\right)  \tag{40}\\
& Q_{\mathrm{H}_{2} \mathrm{O}}=\left\{F_{\mathrm{H}_{2} \mathrm{O}}^{i}+q_{\mathrm{H}_{2} \mathrm{O}}-\beta w_{\mathrm{H}_{2} \mathrm{O}}\left(F^{i}-Q_{\mathrm{EG}}\right)\right\} /\left(1-\beta w_{\mathrm{H}_{2} \mathrm{O}}\right) \tag{41}
\end{align*}
$$

On the assumption that the total pressure in a reactor is obtained by the sum of the partial pressures of EG and water, the vapor phase mole fractions of EG and water are given by the following equations.

$$
\begin{gather*}
y_{\mathrm{EG}}=\frac{Q_{\mathrm{EG}} / 62.07}{Q_{\mathrm{EG}} / 62.07+Q_{\mathrm{H}_{2} \mathrm{O}} / 18.02}  \tag{42}\\
y_{\mathrm{EG}}+y_{\mathrm{H}_{2} \mathrm{O}}=1  \tag{43}\\
x_{\mathrm{EG}} \cdot \gamma_{\mathrm{EG}} \cdot P_{\mathrm{EG}}=y_{\mathrm{EG}} \cdot P  \tag{44}\\
x_{\mathrm{H}_{2} \mathrm{O}} \cdot \gamma_{\mathrm{H}_{2} \mathrm{O}} \cdot P_{\mathrm{H}_{2} \mathrm{O}}=y_{\mathrm{H}_{2} \mathrm{O}} \cdot P  \tag{45}\\
x_{\mathrm{EG}}+x_{\mathrm{H}_{2} \mathrm{O}}+x_{\mathrm{OLG}}=1 \tag{46}
\end{gather*}
$$

where
$x_{\mathrm{EG}}, x_{\mathrm{H}_{2} \mathrm{O}}, x_{\mathrm{OLG}}=$ liquid phase mole fraction of EG, water, oligomer;
$y_{\mathrm{EG}}, y_{\mathrm{H}_{2} \mathrm{O}}=$ vapor phase mole fraction of EG, water;
$P_{\mathrm{EG}}, P_{\mathrm{H}_{2} \mathrm{O}}=$ vapor pressure of EG, water;
$P=$ total pressure (reaction pressure); and
$\gamma_{\mathrm{EG}}, \gamma_{\mathrm{H}_{2} \mathrm{O}}=$ activity coefficient of EG, water.

The activity coefficients are given by Van Laar equation

$$
\begin{align*}
& \ln \gamma_{\mathrm{EG}}=\frac{A}{\left(1+A \cdot x_{\mathrm{EG}} / B \cdot x_{\mathrm{H}_{2} \mathrm{O}}\right)^{2}}  \tag{47}\\
& \ln \gamma_{\mathrm{H}_{2} \mathrm{O}}=\frac{B}{\left(1+B \cdot x_{\mathrm{H}_{2} \mathrm{O}} / A \cdot x_{\mathrm{EG}}\right)^{2}} \tag{48}
\end{align*}
$$

The vapor pressures are expressed by

$$
\begin{align*}
\log P_{\mathrm{EG}} & =7.8808-1957 /(t+193.8)  \tag{49}\\
\log P_{\mathrm{H}_{2} \mathrm{O}} & =7.9668-1668.2 /(t+228) \tag{50}
\end{align*}
$$

where

$$
\begin{aligned}
A, B= & \text { constants of Van Laar equation, } \\
P_{\mathrm{EG}}, P_{\mathrm{H}_{2} \mathrm{O}}= & \text { vapor pressures of } \mathrm{EG} \text { or water at a reaction temperature } \\
& (\mathrm{mm} \mathrm{Hg}), \text { and } \\
t= & \text { reaction temperature }\left({ }^{\circ} \mathrm{C}\right) .
\end{aligned}
$$

The use of eqs. (42)-(50) gives the liquid phase or vapor phase mole fractions and the weight fraction of each component can be obtained from the following relation.

$$
\begin{equation*}
w_{i}=x_{i} M_{i} / \sum_{j=1}^{N} x_{i} M_{j} \tag{51}
\end{equation*}
$$

where subscripts of $i$ and $j$ represent water, EG, and oligomer;
$x_{i}, x_{j}=$ liquid phase mole fraction of component $i, j ;$
$w_{i}=$ liquid phase weight fraction of component $i$;
$N=$ number of components; and
$M_{i}, M_{j}=$ molecular weight of component $i, j$.
The number average molecular weight of oligomer ( $M_{\mathrm{OLG}}$ ) in the liquid phase containing EG and water and excluding undissolved TPA is estimated by

$$
\begin{equation*}
M_{\mathrm{OLG}}=\frac{2000 \beta w_{\mathrm{OLG}}}{\beta C_{1}+\mathrm{OHV}\left[1-\beta\left(w_{\mathrm{EG}}+w_{\mathrm{H}_{2} \mathrm{O}}\right)\right]} \tag{52}
\end{equation*}
$$

The derivation of eq. (52) is given in Appendix C.

## Material Balance in a CSTR

The material balance of each component in the Jth reactor of CSTR is as follows.
Carboxyl end groups

$$
\begin{equation*}
F_{P}^{J-1} \cdot C_{10}-F_{P}^{J} \cdot A V_{a}+\beta W\left(d C_{1} / d \theta\right)=0 \tag{53}
\end{equation*}
$$

Hydroxyl end groups

$$
\begin{equation*}
F_{P}^{J-1} \cdot C_{30}-\beta F_{P}^{J} \cdot C_{3}+\beta W\left(d C_{3} / d \theta\right)=0 \tag{54}
\end{equation*}
$$

## Diester groups

$$
\begin{equation*}
F_{P}^{J-1} \cdot C_{40}-\beta F_{P}^{J} \cdot C_{4}+\beta W\left(d C_{4} / d \theta\right)=0 \tag{55}
\end{equation*}
$$

DEG

$$
\begin{align*}
F_{P}^{J-1} \cdot\left(C_{60}+C_{70}+C_{80}\right)-\beta F_{P}^{J} & \cdot\left(C_{6}+C_{7}+C_{8}\right) \\
& +\beta W\left(d C_{6} / d \theta+d C_{7} / d \theta+d C_{8} / d \theta\right)=0 \tag{56}
\end{align*}
$$

Water

$$
\begin{equation*}
F_{P}^{J-1} \cdot C_{50}+\delta_{\mathbf{H}_{2} \mathrm{O}} \cdot F_{S} \cdot \eta_{\mathrm{H}_{2} \mathrm{O}}-\beta F_{P}^{J} \cdot C_{5}-Q_{\mathrm{H}_{2} \mathrm{O}} \cdot \eta_{\mathrm{H}_{2} \mathrm{O}}+\beta W\left(d C_{5} / d \theta\right)=0 \tag{57}
\end{equation*}
$$

EG

$$
\begin{equation*}
F_{P}^{J-1} \cdot C_{20}+\delta_{\mathrm{EG}} \cdot F_{S} \cdot \eta_{\mathrm{EG}}-\beta F_{P}^{J} \cdot C_{2}-Q_{\mathrm{EG}} \cdot \eta_{\mathrm{EG}}+\beta W\left(d C_{2} / d \theta\right)=0 \tag{58}
\end{equation*}
$$

where

$$
C_{\mathrm{K} 0}=\text { input concentration of component } K \text { into } \mathrm{Jth} \text { reactor; }
$$

$F_{P}^{J}=$ output rate of reaction mixtures from the $J$ th reactor;

$$
\begin{aligned}
F_{S}= & \text { throughput based on PET polymer } ; F_{S}=\frac{166.23}{192.17} \cdot F_{\mathrm{TPA}} ; \\
W= & \text { total weight of reaction mixtures in the Jth reactor; } \\
\delta_{\mathrm{EG}}, \delta_{\mathrm{H}_{2} \mathrm{O}}= & \text { flow rates of EG and water added anew to the Jth reactor per } 1 \\
& \mathrm{~kg} / \mathrm{h} \text { of throughput based on PET polymer; } \\
\gamma_{\mathrm{EG}}, \eta_{\mathrm{H}_{2} \mathrm{O}}= & \text { equivalent concentrations of EG and water; } \eta_{\mathrm{EG}}=16.111 \mathrm{~mol} / \\
& \mathrm{kg}(\mathrm{EG}), \eta_{\mathrm{H}_{2} \mathrm{O}}=55.494 \mathrm{~mol} / \mathrm{kg}\left(\mathrm{H}_{2} \mathrm{O}\right) .
\end{aligned}
$$

## PROCEDURE FOR ESTIMATION OF CONCENTRATION OF EACH COMPONENT IN LIQUID PHASE

The concentration of each component is determined with given values for $A V, O H V$, and $e$ by the procedure shown in Figure 1 entailing:

Step 1. $q_{\mathrm{EG}}$ and $q_{\mathrm{H}_{2} \mathrm{O}}$ are given by eqs. (26), (27), (29), (35), and (36).
Step 2. $x_{i}$ and $M_{\text {OLG }}$ are assumed as an initial value.
Step 3. $w_{i}$ is given by eq. (51), and $\beta$ is given by eqs. (20), (21), and (25).
Step 4. New $M_{\mathrm{OLG}}\left(M_{\mathrm{OLG}, N}\right.$ ) is given by eq. (52)-If the difference between $M_{\text {OLG }, N}$ and $M_{\text {OLG }}$ is very small (less than $\epsilon_{1}$ ), the calculation proceeds to the next step after $M_{\mathrm{OLG}, N}$ is replaced by $M_{\mathrm{OLG}}$. If not so, the calculation returns to Step 3 after $M_{\mathrm{OLG}, N}$ is replaced by $M_{\mathrm{OLG}}$.
Step 5. $Q_{\mathrm{EG}}$ and $Q_{\mathrm{H}_{2} \mathrm{O}}$ are given by eqs. (40) and (41). $y_{\mathrm{EG}}$ is given by eq. (42).

Step 6. New $x_{i}\left(x_{i, N}\right)$ is given by eqs. (43)-(50).-If the difference between $x_{i, N}$ and $x_{i}$ is very small (less than $\epsilon_{2}$ ), $x_{i}$ is determined after $x_{i, N}$ is replaced by $x_{i}$. If not so, the calculation returns to Step 3 after $x_{i, N}$ is replaced by $x_{i}$.
Step 7. $x_{i}, y_{i}, w_{i}, M_{\mathrm{OLG}}, Q_{\mathrm{EG}}, Q_{\mathrm{H}_{2} \mathrm{O}}$, and $\beta$ are determined as results.

## EFFECT OF DIANTIMONY TRIOXIDE AND TITANIUM DIOXIDE ON REACTION RATE CONSTANTS

The reaction rate constants ( $k_{i}$ ) of the reaction scheme for the direct esterification between TPA and EG are related to the reaction temperature and the concentrations of diantimony trioxide and titanium dioxide as follows. ${ }^{4,7}$

$$
\begin{align*}
& k_{1}=\left(3.75 \times 10^{-4} \cdot \mathrm{Sb}^{3+}+1\right) \times 1.5657 \times 10^{9} \cdot \exp (-19640 / R T) \\
& k_{2}=\left(4.75 \times 10^{-4} \cdot \mathrm{Sb}^{3+}+1\right) \times 1.5515 \times 10^{8} \cdot \exp (-18140 / R T) \\
& k_{3}=\left(6.25 \times 10^{-4} \cdot \mathrm{Sb}^{3+}+0.1[\mathrm{TDO}]+1\right) \\
& \times 3.5165 \times 10^{9} \cdot \exp (-22310 / R T)  \tag{61}\\
& k_{4}=\left(4.50 \times 10^{-4} \cdot \mathrm{Sb}^{3+}+1\right) \times 6.7640 \times 10^{7} \cdot \exp (-18380 / R T)  \tag{62}\\
& k_{5}=\left(3.50 \times 10^{-4} \cdot \mathrm{Sb}^{3+}+0.08[\mathrm{TDO}]+1\right) \times 7.7069 \cdot \exp (-2810 / R T)  \tag{63}\\
& k_{6}=\left(1.75 \times 10^{-4} \cdot \mathrm{Sb}^{3+}+0.04[\mathrm{TDO}]+1\right) \\
& \times 6.2595 \times 10^{6} \cdot \exp (-14960 / R T)  \tag{64}\\
& k_{7}=\left(3.75 \times 10^{-4} \cdot \mathrm{Sb}^{3+}+0.085714[\mathrm{TDO}]+1\right) \\
& \times 2.0583 \times 10^{15} \cdot \exp (-42520 / R T) \tag{65}
\end{align*}
$$



Fig. 1. Determination procedure of $x_{i}, y_{i}, w_{i}, M_{\mathrm{OLG}}, Q_{\mathrm{EG}}, Q_{\mathrm{H}_{2} \mathrm{O}}$, and $\beta$ in use of $A V, O H V$, and $e$.
where $\mathrm{Sb}^{3+}$ is the mole number of antimony ion ( $\mathrm{Sb}^{3+}$ ) in diantimony trioxide supplied for $10^{6}$ moles of TPA; [TDO] is the weight percent of charged titanium dioxide against PET; $R$ is the gas constant; $T$ is the absolute temperature.

## RESULTS AND DISCUSSION

Table I lists the simulation results at various concentrations of water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) added the first esterification reactor under the conditions of $P=0,0.5$
and $1.0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G}, \mathrm{E} / \mathrm{T}=2.0, \tau=5.0 \mathrm{~h}$, concentration of antimony trioxide ( $\mathrm{Sb}^{3+}$ ) $=400 \mu \mathrm{~mol} /$ TPAmol, and concentration of titanium dioxide ([TDO]) $=0.5 \mathrm{wt} \% /$ PET. Figures $2-7$ show the relations of the amount of water added to the first reactor ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) and the dried oligomer properties ( $A V, \phi, P_{n}, M_{n}$, $\left.E_{S}, d\right)$ at $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G}$ with the simulation results tabulated in Table I. In general, these figures suggest that the higher the $\delta_{\mathrm{H}_{2} \mathrm{O}}$ is, the lower the reaction rates become and then the higher the reaction temperature is, the higher the degree of decrease in the reaction rates becomes. This is due to why the higher $\delta_{\mathrm{H}_{2} \mathrm{O}}$ brings about the higher decrease of EG dissolved in the reaction mixtures by the vapor-liquid equilibrium relation in the reactor: Although the higher $\delta_{\mathrm{H}_{2} \mathrm{O}}$ gives the worse results, the main reaction rates become lower, as shown in Figures 2-6; on the other hand, it brings the better results that the side reaction of diethylene glycol (DEG) formation is suppressed, as shown in Figure 7.

Figure 8 shows the relation between $\delta_{\mathrm{H}_{2} \mathrm{O}}$ and the weight fraction of water in the vapor phase (distillate), $w_{\mathrm{H}_{2} \mathrm{O}}^{d}$. In spite of addition of water, $w_{\mathrm{H}_{2} \mathrm{O}}^{d}$ becomes lower as $\delta_{\mathrm{H}_{2} \mathrm{O}}$ becomes higher. From this figure that is considered why EG in the reaction mixtures decreases so as to satisfy the vapor-liquid equilibrium by adding water to the reactor, so that EG removed from the reaction mixtures brings its increase in the vapor phase. This fact means that by the adequate addition of water polymer of low DEG content can be produced.

Figure 9 shows the relation between $A V$ and $\beta$. From this figure, it is confirmed that this relation can be expressed by a fixed curve, independent of $\delta_{\mathrm{H}_{2} \mathrm{O}}$ and reaction temperature. This figure suggests that the clearing point where solid TPA begins not to exist in the reaction mixtures, that is the point of $\beta$ $=1$, is the point of $A V=0.6 \mathrm{eq} / \mathrm{kg}$. This gives almost the same result as that obtained from our experimental observation of the pilot plant. The agreement of the estimated and observed results suggests support of the validity of the present model; the mathematical model, solubilities of TPA, assumption of vapor-liquid equilibrium as ideal solution, and so on.

## CONCLUSION

The computer simulation was carried out to predict the effect of water in EG added into a reactor and on the esterification reactions. From the simulation results, it has been quantitatively understood that the addition of water brings about the undesirable effect of the decrease in main reactions in the esterification process, but on the other hand it gives the desirable effect of the suppression of side reactions (DEG formation reactions). It has also been shown that the reaction pressure has a great influence on their reactions.

The good agreement of the clearing point between estimated and observed results supports the validity of the present model.

$$
\text { APPENDIX A: DERIVATION OF } k_{8}=2 k_{7} \text { AND } k_{9}=4 k_{7}
$$

[^0]TABLE I
Simulation Results for First Esterification Reactor at

| $\begin{gathered} t \\ \left({ }^{\circ} \mathrm{C}\right) \end{gathered}$ | $\begin{gathered} P \\ \left(\mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G}\right) \end{gathered}$ | (h) | $E / T$ | $\delta_{\text {EG }}$ | $\delta_{\mathrm{H}_{2} \mathrm{O}}$ | $\begin{gathered} \text { AV } \\ (\mathrm{eq} / \mathrm{kg}) \end{gathered}$ | $\begin{gathered} \mathrm{OHV} \\ (\mathrm{eq} / \mathrm{kg}) \end{gathered}$ | $\phi$ | $E$ s, | $M_{n}$ | $P_{n}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 240 | 0.0 | 5.0 | 2.0 | 0.00 | 0.00 | 4.583 | 1.294 | 0.220 | 0.568 | 340 | 1.804 |
| 240 | 0.0 | 5.0 | 2.0 | 0.00 | 0.01 | 5.155 | 1.216 | 0.191 | 0.519 | 314 | 1.681 |
| 240 | 0.0 | 5.0 | 2.0 | 0.00 | 0.02 | 5.785 | 1.128 | 0.163 | 0.465 | 289 | 1.565 |
| 240 | 0.0 | 5.0 | 2.0 | 0.00 | 0.03 | 6.499 | 1.025 | 0.136 | 0.407 | 266 | 1.456 |
| 240 | 0.0 | 5.0 | 2.0 | 0.00 | 0.04 | 7.384 | 0.894 | 0.108 | 0.336 | 242 | 1.343 |
| 240 | 0.0 | 5.0 | 2.0 | 0.00 | 0.05 | 8.750 | 0.676 | 0.072 | 0.231 | 212 | 1.207 |
| 250 | 0.0 | 5.0 | 2.0 | 0.00 | 0.00 | 2.447 | 1.186 | 0.327 | 0.763 | 550 | 2.847 |
| 250 | 0.0 | 5.0 | 2.0 | 0.00 | 0.01 | 2.897 | 1.149 | 0.284 | 0.722 | 494 | 2.575 |
| 250 | 0.0 | 5.0 | 2.0 | 0.00 | 0.02 | 3.373 | 1.108 | 0.247 | 0.679 | 446 | 2.343 |
| 250 | 0.0 | 5.0 | 2.0 | 0.00 | 0.03 | 3.884 | 1.063 | 0.215 | 0.633 | 404 | 2.139 |
| 250 | 0.0 | 5.0 | 2.0 | 0.00 | 0.04 | 4.434 | 1.013 | 0.186 | 0.585 | 367 | 1.959 |
| 250 | 0.0 | 5.0 | 2.0 | 0.00 | 0.05 | 5.052 | 0.955 | 0.159 | 0.531 | 333 | 1.794 |
| 260 | 0.0 | 5.0 | 2.0 | 0.00 | 0.00 | 0.600 | 0.930 | 0.608 | 0.941 | 1306 | 6.648 |
| 260 | 0.0 | 5.0 | 2.0 | 0.00 | 0.01 | 0.961 | 0.914 | 0.488 | 0.906 | 1067 | 5.457 |
| 260 | 0.0 | 5.0 | 2.0 | 0.00 | 0.02 | 1.338 | 0.897 | 0.401 | 0.870 | 895 | 4.603 |
| 260 | 0.0 | 5.0 | 2.0 | 0.00 | 0.03 | 1.707 | 0.879 | 0.340 | 0.835 | 773 | 4.001 |
| 260 | 0.0 | 5.0 | 2.0 | 0.00 | 0.04 | 2.148 | 0.858 | 0.286 | 0.794 | 665 | 3.463 |
| 260 | 0.0 | 5.0 | 2.0 | 0.00 | 0.05 | 2.583 | 0.837 | 0.245 | 0.753 | 585 | 3.063 |
| 240 | 0.5 | 5.0 | 2.0 | 0.00 | 0.00 | 2.357 | 1.874 | 0.443 | 0.767 | 473 | 2.389 |
| 240 | 0.5 | 5.0 | 2.0 | 0.00 | 0.01 | 2.739 | 1.821 | 0.399 | 0.731 | 439 | 2.232 |
| 240 | 0.5 | 5.0 | 2.0 | 0.00 | 0.02 | 3.135 | 1.765 | 0.360 | 0.694 | 408 | 2.092 |
| 240 | 0.5 | 5.0 | 2.0 | 0.00 | 0.03 | 3.558 | 1.704 | 0.324 | 0.655 | 380 | 1.962 |
| 240 | 0.5 | 5.0 | 2.0 | 0.00 | 0.04 | 4.005 | 1.639 | 0.290 | 0.615 | 354 | 1.844 |
| 240 | 0.5 | 5.0 | 2.0 | 0.00 | 0.05 | 4.482 | 1.567 | 0.259 | 0.573 | 331 | 1.735 |
| 250 | 0.5 | 5.0 | 2.0 | 0.00 | 0.00 | 0.481 | 1.607 | 0.770 | 0.952 | 958 | 4.758 |
| 250 | 0.5 | 5.0 | 2.0 | 0.00 | 0.01 | 0.687 | 1.575 | 0.696 | 0.931 | 884 | 4.409 |
| 250 | 0.5 | 5.0 | 2.0 | 0.00 | 0.02 | 1.014 | 1.550 | 0.604 | 0.899 | 780 | 3.911 |
| 250 | 0.5 | 5.0 | 2.0 | 0.00 | 0.03 | 1.350 | 1.522 | 0.530 | 0.866 | 696 | 3.509 |
| 250 | 0.5 | 5.0 | 2.0 | 0.00 | 0.04 | 1.702 | 1.493 | 0.467 | 0.832 | 626 | 3.173 |
| 250 | 0.5 | 5.0 | 2.0 | 0.00 | 0.05 | 2.066 | 1.461 | 0.414 | 0.797 | 567 | 2.891 |
| 260 | 0.5 | 5.0 | 2.0 | 0.00 | 0.00 | 0.394 | 1.421 | 0.783 | 0.960 | 1102 | 5.490 |
| 260 | 0.5 | 5.0 | 2.0 | 0.00 | 0.01 | 0.415 | 1.372 | 0.768 | 0.958 | 1119 | 5.590 |
| 260 | 0.5 | 5.0 | 2.0 | 0.00 | 0.02 | 0.436 | 1.326 | 0.752 | 0.956 | 1135 | 5.681 |
| 260 | 0.5 | 5.0 | 2.0 | 0.00 | 0.03 | 0.458 | 1.281 | 0.737 | 0.954 | 1150 | 5.765 |
| 260 | 0.5 | 5.0 | 2.0 | 0.00 | 0.04 | 0.480 | 1.240 | 0.721 | 0.952 | 11.63 | 5.840 |
| 260 | 0.5 | 5.0 | 2.0 | 0.00 | 0.05 | 0.503 | 1.201 | 0.705 | 0.950 | 1173 | 5.905 |
| 240 | 1.0 | 5.0 | 2.0 | 0.00 | 0.00 | 1.383 | 2.280 | 0.622 | 0.859 | 546 | 2.686 |
| 240 | 1.0 | 5.0 | 2.0 | 0.00 | 0.01 | 1.684 | 2.236 | 0.570 | 0.830 | 510 | 2.523 |
| 240 | 1.0 | 5.0 | 2.0 | 0.00 | 0.02 | 1.996 | 2.191 | 0.523 | 0.799 | 478 | 2.376 |
| 240 | 1.0 | 5.0 | 2.0 | 0.00 | 0.03 | 2.320 | 2.142 | 0.480 | 0.768 | 448 | 2.243 |
| 240 | 1.0 | 5.0 | 2.0 | 0.00 | 0.04 | 2.657 | 2.091 | 0.440 | 0.736 | 421 | 2.121 |
| 240 | 1.0 | 5.0 | 2.0 | 0.00 | 0.05 | 3.003 | 2.037 | 0.404 | 0.704 | 397 | 2.011 |
| 250 | 1.0 | 5.0 | 2.0 | 0.00 | 0.00 | 0.418 | 2.020 | 0.828 | 0.957 | 820 | 4.009 |
| 250 | 1.0 | 5.0 | 2.0 | 0.00 | 0.01 | 0.436 | 1.967 | 0.819 | 0.955 | 832 | 4.077 |
| 250 | 1.0 | 5.0 | 2.0 | 0.00 | 0.02 | 0.454 | 1.917 | 0.808 | 0.954 | 843 | 4.140 |
| 250 | 1.0 | 5.0 | 2.0 | 0.00 | 0.03 | 0.473 | 1.869 | 0.798 | 0.952 | 854 | 4.200 |
| 250 | 1.0 | 5.0 | 2.0 | 0.00 | 0.04 | 0.587 | 1.857 | 0.760 | 0.941 | 819 | 4.036 |
| 250 | 1.0 | 5.0 | 2.0 | 0.00 | 0.05 | 0.751 | 1.800 | 0.706 | 0.924 | 784 | 3.881 |
| 260 | 1.0 | 5.0 | 2.0 | 0.00 | 0.00 | 0.337 | 1.838 | 0.845 | 0.966 | 919 | 4.502 |
| 260 | 1.0 | 5.0 | 2.0 | 0.00 | 0.01 | 0.352 | 1.782 | 0.835 | 0.964 | 937 | 4.599 |
| 260 | 1.0 | 5.0 | 2.0 | 0.00 | 0.02 | 0.368 | 1.730 | 0.825 | 0.963 | 954 | 4.692 |
| 260 | 1.0 | 5.0 | 2.0 | 0.00 | 0.03 | 0.384 | 1.679 | 0.814 | 0.961 | 969 | 4.780 |
| 260 | 1.0 | 5.0 | 2.0 | 0.00 | 0.04 | 0.400 | 1.631 | 0.803 | 0.960 | 985 | 4.866 |
| 260 | 1.0 | 5.0 | 2.0 | 0.00 | 0.05 | 0.417 | 1.586 | 0.792 | 0.958 | 999 | 4.945 |

$F_{s}=100 \mathrm{~kg} / \mathrm{h}, \mathrm{Sb}_{2} \mathrm{O}_{3}=400 \mu \mathrm{~mol}, \mathrm{~T}_{\mathrm{i}} \mathrm{O}_{2}=0.5 \mathrm{wt} \%$.

Various Amounts of Water Charged into Reactor

| $\begin{gathered} 100 e \\ (\%) \end{gathered}$ | $\begin{gathered} d \\ (\%) \end{gathered}$ | $w_{\text {EG }}$ | $w_{\mathrm{H}_{2} \mathrm{O}}$ | $w_{\mathrm{H}_{2} \mathrm{O}}^{d}$ | $\begin{gathered} Q \\ (\mathrm{~kg} / \mathrm{h}) \end{gathered}$ | $\begin{gathered} \boldsymbol{F}_{\mathbf{P}} \\ (\mathrm{kg} / \mathrm{h}) \end{gathered}$ | $\begin{gathered} F_{P}^{o} \\ (\mathrm{~kg} / \mathrm{h}) \end{gathered}$ | $\beta$ | $\begin{gathered} T_{m} \\ \left({ }^{\circ} \mathrm{C}\right) \end{gathered}$ | $\beta w_{E G}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.891 | 0.609 | 0.0158 | 0.0004 | 0.205 | 51.9 | 99.1 | 98.1 | 0.646 | 226 | 0.0102 |
| 0.936 | 0.586 | 0.0164 | 0.0004 | 0.199 | 53.9 | 98.1 | 97.1 | 0.597 | 224 | 0.0098 |
| 0.986 | 0.556 | 0.0171 | 0.0004 | 0.191 | 56.0 | 97.0 | 96.1 | 0.543 | 215 | 0.0093 |
| 1.055 | 0.522 | 0.0180 | 0.0004 | 0.182 | 58.2 | 95.8 | 95.0 | 0.482 | - | 0.0087 |
| 1.162 | 0.478 | 0.0194 | 0.0004 | 0.170 | 60.7 | 94.4 | 93.6 | 0.406 | - | 0.0079 |
| 1.385 | 0.396 | 0.0223 | 0.0004 | 0.146 | 63.9 | 92.1 | 91.5 | 0.288 | - | 0.0064 |
| 0.922 | 0.802 | 0.0072 | 0.0003 | 0.287 | 49.9 | 101.1 | 100.5 | 0.831 | 229 | 0.0060 |
| 0.959 | 0.791 | 0.0075 | 0.0003 | 0.281 | 51.6 | 100.4 | 99.8 | 0.793 | 231 | 0.0059 |
| 1.000 | 0.777 | 0.0077 | 0.0003 | 0.276 | 53.4 | 99.6 | 99.0 | 0.752 | 234 | 0.0058 |
| 1.050 | 0.762 | 0.0080 | 0.0003 | 0.269 | 55.2 | 98.8 | 98.3 | 0.708 | 238 | 0.0057 |
| 1.107 | 0.744 | 0.0083 | 0.0003 | 0.262 | 57.1 | 98.0 | 97.4 | 0.660 | 241 | 0.0055 |
| 1.179 | 0.722 | 0.0087 | 0.0003 | 0.254 | 59.0 | 97.0 | 96.5 | 0.607 | 240 | 0.0053 |
| 0.830 | 0.850 | 0.0035 | 0.0002 | 0.363 | 48.5 | 102.5 | 102.1 | 0.996 | 237 | 0.0035 |
| 0.858 | 0.847 | 0.0035 | 0.0002 | 0.359 | 50.1 | 102.0 | 101.6 | 0.964 | 234 | 0.0034 |
| 0.892 | 0.847 | 0.0036 | 0.0002 | 0.354 | 51.6 | 101.4 | 101.0 | 0.932 | 233 | 0.0034 |
| 0.918 | 0.837 | 0.0037 | 0.0002 | 0.351 | 53.2 | 100.9 | 100.5 | 0.900 | 232 | 0.0033 |
| 0.965 | 0.837 | 0.0038 | 0.0002 | 0.344 | 54.8 | 100.2 | 99.9 | 0.861 | 233 | 0.0033 |
| 1.011 | 0.834 | 0.0039 | 0.0002 | 0.339 | 56.5 | 99.6 | 99.2 | 0.824 | 236 | 0.0032 |
| 1.074 | 1.011 | 0.0201 | 0.0008 | 0.309 | 46.4 | 104.6 | 102.8 | 0.838 | 215 | 0.0168 |
| 1.109 | 0.998 | 0.0205 | 0.0008 | 0.304 | 48.1 | 103.9 | 102.1 | 0.805 | 216 | 0.0165 |
| 1.149 | 0.984 | 0.0211 | 0.0008 | 0.300 | 49.9 | 103.1 | 101.4 | 0.772 | 216 | 0.0163 |
| 1.192 | 0.967 | 0.0217 | 0.0008 | 0.295 | 51.7 | 102.3 | 100.7 | 0.736 | 217 | 0.0160 |
| 1.245 | 0.950 | 0.0224 | 0.0008 | 0.289 | 53.5 | 101.5 | 99.9 | 0.698 | 217 | 0.0156 |
| 1.301 | 0.928 | 0.0232 | 0.0008 | 0.283 | 55.4 | 100.6 | 99.0 | 0.657 | 216 | 0.0152 |
| 1.124 | 1.237 | 0.0090 | 0.0006 | 0.391 | 45.5 | 105.6 | 104.5 | 1.000 | 225 | 0.0090 |
| 1.124 | 1.210 | 0.0090 | 0.0006 | 0.392 | 46.9 | 105.1 | 104.1 | 0.983 | 224 | 0.0088 |
| 1.159 | 1.207 | 0.0092 | 0.0006 | 0.388 | 48.5 | 104.6 | 103.6 | 0.955 | 223 | 0.0088 |
| 1.199 | 1.205 | 0.0094 | 0.0006 | 0.384 | 50.0 | 104.0 | 103.0 | 0.926 | 222 | 0.0087 |
| 1.240 | 1.199 | 0.0097 | 0.0006 | 0.379 | 51.6 | 103.4 | 102.5 | 0.896 | 222 | 0.0087 |
| 1.285 | 1.194 | 0.0099 | 0.0006 | 0.374 | 53.2 | 102.8 | 101.9 | 0.865 | 223 | 0.0086 |
| 1.834 | 1.987 | 0.0061 | 0.0004 | 0.388 | 46.3 | 104.7 | 104.0 | 1.000 | 229 | 0.0061 |
| 1.718 | 1.851 | 0.0059 | 0.0004 | 0.398 | 47.5 | 104.5 | 103.8 | 1.000 | 230 | 0.0059 |
| 1.612 | 1.727 | 0.0056 | 0.0004 | 0.408 | 48.8 | 104.3 | 103.7 | 1.000 | 231 | 0.0056 |
| 1.514 | 1.614 | 0.0054 | 0.0004 | 0.418 | 50.0 | 104.1 | 103.5 | 1.000 | 231 | 0.0054 |
| 1.426 | 1.512 | 0.0052 | 0.0004 | 0.427 | 51.1 | 103.9 | 103.3 | 1.000 | 232 | 0.0052 |
| 1.346 | 1.420 | 0.0050 | 0.0004 | 0.435 | 52.3 | 103.7 | 103.2 | 1.000 | 232 | 0.0050 |
| 1.305 | 1.406 | 0.0252 | 0.0014 | 0.373 | 42.8 | 108.2 | 105.6 | 0.921 | 209 | 0.0232 |
| 1.341 | 1.397 | 0.0257 | 0.0014 | 0.370 | 44.5 | 107.6 | 105.0 | 0.896 | 210 | 0.0230 |
| 1.381 | 1.389 | 0.0263 | 0.0014 | 0.366 | 46.1 | 106.9 | 104.4 | 0.870 | 210 | 0.0229 |
| 1.423 | 1.378 | 0.0269 | 0.0014 | 0.362 | 47.8 | 106.3 | 103.8 | 0.842 | 210 | 0.0226 |
| 1.469 | 1.367 | 0.0275 | 0.0014 | 0.358 | 49.4 | 105.6 | 103.1 | 0.814 | 210 | 0.0224 |
| 1.518 | 1.353 | 0.0282 | 0.0014 | 0.354 | 51.1 | 104.9 | 102.5 | 0.785 | 210 | 0.0221 |
| 1.762 | 2.015 | 0.0138 | 0.0010 | 0.411 | 43.4 | 107.6 | 106.1 | 1.000 | 218 | 0.0138 |
| 1.675 | 1.905 | 0.0132 | 0.0009 | 0.421 | 44.7 | 107.4 | 105.8 | 1.000 | 218 | 0.0132 |
| 1.595 | 1.803 | 0.0126 | 0.0009 | 0.431 | 45.9 | 107.1 | 105.6 | 1.000 | 219 | 0.0126 |
| 1.521 | 1.711 | 0.0121 | 0.0009 | 0.440 | 47.2 | 106.8 | 105.5 | 1.000 | 220 | 0.0121 |
| 1.472 | 1.638 | 0.0120 | 0.0009 | 0.445 | 48.4 | 106.6 | 105.2 | 0.992 | 220 | 0.0119 |
| 1.487 | 1.620 | 0.0118 | 0.0009 | 0.446 | 49.9 | 106.1 | 104.8 | 0.978 | 220 | 0.0115 |
| 3.015 | 3.375 | 0.0096 | 0.0007 | 0.406 | 44.4 | 106.6 | 105.5 | 1.000 | 222 | 0.0096 |
| 2.845 | 3.169 | 0.0092 | 0.0007 | 0.416 | 45.7 | 106.4 | 105.3 | 1.000 | 222 | 0.0092 |
| 2.689 | 2.981 | 0.0087 | 0.0007 | 0.426 | 46.9 | 106.1 | 105.1 | 1.000 | 223 | 0.0087 |
| 2.544 | 2.807 | 0.0083 | 0.0007 | 0.435 | 48.2 | 105.9 | 104.9 | 1.000 | 224 | 0.0083 |
| 2.410 | 2.646 | 0.0080 | 0.0007 | 0.444 | 49.4 | 105.7 | 104.7 | 1.000 | 225 | 0.0080 |
| 2.287 | 2.500 | 0.0076 | 0.0007 | 0.453 | 50.6 | 105.4 | 104.6 | 1.000 | 226 | 0.0076 |



Fig. 2. Influence of added water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) on $A V$ under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G} . \bullet, 240^{\circ} \mathrm{C} ; \Delta, 250^{\circ} \mathrm{C}$; $\mathbf{m}$, $260^{\circ} \mathrm{C}$.

For $k_{7}$ in eq. (4), its combination number is one from the following expression


For $k_{8}$ in eq. (5), its combination number is two from the following expression




Fig. 3. Influence of added water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) on $\phi$ under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G} . \bullet, 240^{\circ} \mathrm{C} ; \mathbf{\Delta}, 250^{\circ} \mathrm{C}$; $\boldsymbol{m}$, $260^{\circ} \mathrm{C}$.


Fig. 4. Influence of added water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) on $P_{n}$ under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G} . \bullet, 240^{\circ} \mathrm{C} ; \Delta, 250^{\circ} \mathrm{C} ; \boldsymbol{\square}$, $260^{\circ} \mathrm{C}$.


Fig. 5. Influence of added water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) on $M_{n}$ under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G} . \bullet, 240^{\circ} \mathrm{C} ; \Delta, 250^{\circ} \mathrm{C} ; \boldsymbol{\square}$, $260^{\circ} \mathrm{C}$.


Fig. 6. Influence of added water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) on $E_{S}$ under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G} . \bullet, 240^{\circ} \mathrm{C} ; \mathbf{\Delta}, 250^{\circ} \mathrm{C}$; $\boldsymbol{\square}$, $260^{\circ} \mathrm{C}$.

For $k_{9}$ in eq. (6), its combination number is four from the following expression


If the hydroxyl end groups have the same reactivity, it can be derived from the above combinations that $k_{8}=2 k_{7}$ and $k_{9}=4 k_{7}$.


Fig. 7. Influence of added water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) on $d$ under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G} . \bullet, 240^{\circ} \mathrm{C} ; \Delta, 250^{\circ} \mathrm{C}$; $\mathbf{\square}$, $260^{\circ} \mathrm{C}$.


Fig. 8. Relation between added water ( $\delta_{\mathrm{H}_{2} \mathrm{O}}$ ) and weight fraction of water in the vapor phase ( $w_{\mathrm{H}_{2} \mathrm{O}}^{d}$ ) under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G} . \bullet, 240^{\circ} \mathrm{C} ; \Delta, 250^{\circ} \mathrm{C} ; \boxminus, 260^{\circ} \mathrm{C}$.

## APPENDIX B: DERIVATION OF EQS. (35) AND (36)

## Derivation of Eq. (35) ( $\boldsymbol{q}_{\mathrm{EG}}$ )

We consider a molecule of oligomer of poly (ethylene terephthalate) (PET) with the polymerization degree of $P_{n}$. The amount of ethylene glycol (EG) bound into the polymer chain is given by $\left(P_{n}-1\right) /(1+e)$, and that of the end group given by $2 \phi /(1+e)$. Then the amount of EG due to ether (DEG) linkage bound into the polymer chain is given by $2\left(P_{n}-1\right) e /(1+e)$, and that of the end group given by $4 \phi e /(1+e)$.

The ratio of bound EG to a polymer chain linkage is given by $\left[\left(P_{n}-1\right) /(1+e)+2 \phi /(1+e)\right.$ $\left.+2\left(P_{n}-1\right) e /(1+e)+4 \phi e /(1+e)\right] / P_{n}$.

From the above, the changed amount of bound EG ( $q_{\mathrm{EG}}$ ) from the polymerization degree of $P_{\mathrm{ni}}$ to $P_{\mathrm{nf}}$, which is equal to that consumed by the reactions shown in eqs. (1)-(7), is expressed by the following equation


Fig. 9. Relation between $A V$ and weight fraction of liquid phase $(\beta)$ under $P=0 \mathrm{~kg} / \mathrm{cm}^{2} \mathrm{G}$. $\bullet, 240^{\circ} \mathrm{C} ; \mathbf{\Delta}, 250^{\circ} \mathrm{C}$; $\boldsymbol{\mathrm { m }}, 260^{\circ} \mathrm{C}$.

$$
\begin{align*}
& q_{\mathrm{EG}}= F_{\mathrm{TPA}}\left(M_{\mathrm{EG}} / M_{\mathrm{EG}}\right)\left\{\left[\left(P_{\mathrm{nf}}-1\right) /(1+e)\right.\right. \\
& \begin{aligned}
&(1+e)+4 \phi_{f} /(1+e)+2\left(P_{\mathrm{nf}}-1\right) e / \\
&\left.+2\left(P_{\mathrm{ni}}-1\right) e /(1+e)\right] / P_{\mathrm{nf}}-\left[\left(P_{\mathrm{ni}}-1\right) /(1+e)+2 \phi_{i} /(1+e)\right. \\
&\left.\left.4 \phi_{i} e /(1+e)\right] / P_{\mathrm{ni}}\right\}
\end{aligned}
\end{align*}
$$

where $M_{\mathrm{EG}}=$ molecular weight of $\mathrm{EG}, M_{\mathrm{EG}}=62.07$; and $M_{\mathrm{TPA}}=$ molecular weight of TPA, $M_{\mathrm{EG}}$ $=166.13$.

Judging from the DEG content in oligomer of a few percents, the influence of DEG on $q_{\text {EG }}$ is negligible. If so ( $e=0$ ), eq. (66) reduces to eq. (35).

## Derivation of Eq. (36) ( $\boldsymbol{q}_{\mathbf{H}_{\mathbf{2}} \mathrm{O}}$ )

After the manner of the derivation of $q_{E G}$, we consider a molecule of PET oligomer with the polymerization degree of $P_{n}$. When a polymer chain linkage is formed by the reactions shown in eqs. (1) and (2), two molecules of water are generated. Therefore, for the formation of the oligomer with the polymer chain linkages of ( $P_{n}-1$ ), the water molecules of $2\left(P_{n}-1\right)$ are generated. Then, as the formation of a hydroxyl end group brings about the generation of a molecule of water, the water molecules of $2 \phi$ are generated for the formation of the oligomer with the hydroxyl end group ratio of $\phi$.

On the other hand, for the formation of an ether (DEG) linkage bound into the polymer chain, three molecules of water are generated from eq. (4); for the formation of the DEG end group, two molecules are generated from eqs. (5)-(7). An oligomer with the polymerization degree of $P_{n}$ comprises the polymer chains with the EG linkages of $\left(P_{n}-1\right) /(1+e)$ and DEG linkages of ( $\left.P_{n}-1\right) e /(1+e)$, and the bifunctional end groups with the carboxyl end groups of $2(1-\phi)$ and hydroxyl endgroups of $2 \phi$; the hydroxyl end groups comprise EG molecules of $2 \phi /(1+e)$ and the DEG molecules of $2 \phi e /(1+e)$.

From the above results, the amount of water ( $q_{\mathrm{H}_{2} \mathrm{O}}$ ) generated by the reactions is expressed by the following equation after the manner of the derivation of $q_{\mathrm{EG}}$.

$$
\left.\left.\begin{array}{rl}
q_{\mathrm{H}_{2} \mathrm{O}}= & F_{\mathrm{TPA}}\left(M_{\mathrm{EG}} / M_{\mathrm{EG}}\right)\left\{\left[2\left(P_{\mathrm{nf}}-1\right) /(1+e)+2 \phi_{f} /(1+e)\right.\right. \\
& +3\left(P_{\mathrm{nf}}-1\right) e /(1+e)
\end{array}\right)+4 \phi_{f} e /(1+e)\right] / P_{\mathrm{nf}}-\left[2\left(P_{\mathrm{ni}}-1\right) /(1+e)\right\}
$$

Judging from the DEG content in oligomer of a few percents, the influence of DEG on $q_{\mathrm{H}_{2} \mathrm{O}}$ is negligible. If so ( $e=0$ ), eq. (67) reduces to eq. (36).

## APPENDIX C: DERIVATION OF EQ. (52)

Now, we consider the reaction mixtures of $W \mathrm{~kg}$ which consist of the liquid phase ( $\beta W \mathrm{~kg}$; oligomer without undissolved TPA and with dissolved TPA, EG, and water) and the solid phase $((1-\beta) W \mathrm{~kg}$; undissolved TPA).

As the weight of oligomer in the liquid phase is $\beta W w_{\text {OLG }} \mathrm{kg}$, the number of molecules of oligomer with the number average molecular weight of $M_{\text {OLG }}$ in the liquid phase is expressed by $\beta W w_{\text {OLG }} /$ ( $M_{\text {oLG }} / 1000$ ). On the other hand, the number of the oligomer molecules can be also given by $\left\{\beta W C_{1}+O H V\left[1-\beta\left(w_{\mathrm{EG}}+w_{\mathrm{H}_{2} \mathrm{O}}\right)\right]\right\} / 2$. From the above results, we can obtain the following relation.

$$
\begin{equation*}
\beta W w_{\mathrm{OLG}} /\left(M_{\mathrm{OLG}} / 1000\right)=\left\{\beta W C_{1}+O H V\left[1-\beta\left(w_{\mathrm{EG}}+w_{\mathrm{H}_{2} \mathrm{O}}\right)\right]\right\} / 2 \tag{68}
\end{equation*}
$$

Rearrangement of eq. (68) gives eq. (52).

## NOMENCLATURE

| AV | concentrations of total carboxyl end groups in dried oligomer, eq $/ \mathrm{kg} ; A V=C_{T} /\left\{T_{S}\right.$ $\left.+W_{e}\left[1-\left(w_{\mathrm{EG}}+w_{\mathrm{H}_{2} \mathrm{O}}\right)\right]\right\}$ |
| :---: | :---: |
| $A V_{a}$ | concentration of total carboxyl end groups in the reaction mixtures, eq/kg; $A V_{a}=C_{T} /$ $\left(T_{S}+W_{e}\right)$ |
| $C_{1}$ | concentration of carboxyl end groups in the liquid phase of reaction mixtures, $\mathrm{eq} / \mathrm{kg}$ |
| $\mathrm{C}_{2}$ | concentration of EG in the liquid phase of reaction mixtures, mol/kg |
| $\mathrm{C}_{3}$ | concentration of hydroxyl end groups in the liquid phase of reaction mixtures, eq/ kg |
| $C_{4}$ | concentration of diester groups in the liquid phase of reaction mixtures, mol $/ \mathrm{kg}$ |
| $\mathrm{C}_{5}$ | concentration of water in the liquid phase of reaction mixtures, $\mathrm{mol} / \mathrm{kg}$ |
| $C_{6}, C_{7}, C_{8}$ | concentration of free and bound DEG in the liquid phase of reaction mixtures, mol/ kg |
| $C_{k 0}$ | input concentration of component $k$, eq/ kg or $\mathrm{mol} / \mathrm{kg}$ |
| $C_{T}$ | total amount of carboxyl end groups, equiv. |
| $d C_{1} / d \theta$ | reaction rate of component $i, \mathrm{eq} / \mathrm{kg} \cdot \mathrm{h}$ or mol $/ \mathrm{kg} \cdot \mathrm{h}$ |
| $d$ | percentage of DEG content in dried oligomer based on bound TPA, $10^{-2} \mathrm{~mol} / \mathrm{mol}$ (TPA) |
| $e$ | molar ratio of DEG content to bound EG in dried oligomer, mol/mol (bound EG) |
| $E_{S}$ | esterification degrees of dried oligomer |
| $E / T$ | molar ratio of EG to TPA of the slurry, which fed to the first reactor |
| $F_{P}$ | actual output rates of reaction mixtures discharged from the reactor, $\mathrm{kg} / \mathrm{h}$ |
| $F_{P}^{J}$ | actual output rates of reaction mixtures discharged from the Jth reactor, $\mathrm{kg} / \mathrm{h}$ |
| $F_{S}$ | throughput based on PET, kg/h |
| $F_{\text {TPA }}$ | throughput based on TPA, $\mathrm{kg} / \mathrm{h}$ |
| $k_{1}$ to $k_{9}$ | reaction rate constants as defined in eqs. (1)-(7) |
| $M_{n}$ | number average molecular weight of oligomer, g/mol |
| $M_{\text {OLG }}$ | number average molecular weight of oligomer in the liquid phase free from undissolved TPA defined as eq. (52), $\mathrm{g} / \mathrm{mol}$ |
| $n$ | number of reactors |
| OHV | concentrations of hydroxyl end groups in 1 kg dried oligomer, eq/ kg |
| $P$ | total pressures ( $=$ reaction pressures), mm Hg or Torr |
| $P_{n}$ | number average degree of polymerization of oligomer |
| $Q$ | total vapor flow rate, $\mathrm{kg} / \mathrm{h} ; \mathrm{Q}=Q_{\mathrm{EGG}}+Q_{\mathrm{H}_{2} \mathrm{O}}$ |
| $Q_{\text {EG }}$ | vapor flow rate of EG, $\mathrm{kg} / \mathrm{h}$ |
| $Q_{\mathrm{H}_{2} \mathrm{O}}$ | vapor flow rate of water, $\mathrm{kg} / \mathrm{h}$ |
| $\mathbf{S b}^{3+}$ | mole number of antimony ion ( $\mathrm{Sb}^{3+}$ ) in diantimony trioxide supplied for $10^{6}$ moles of TPA, $\mu \mathrm{mol} /$ TPAmol |
| SV | saponification value calculated from eq. (30), eq/kg |
| $t$ | reaction temperatures, ${ }^{\circ} \mathrm{C}$ |
| $T_{\text {m }}$ | melting point of dried oligomer, ${ }^{\circ} \mathrm{C}$ |
| [TDO] | weight percent of charged titanium dioxide agianst PET, wt \% |
| $w_{\text {EG }}$ | weight fraction of EG in the liquid phase of reaction mixtures free from undissolved TPA |
| $w_{\mathrm{H}_{2} \mathrm{O}}$ | weight fraction of water in the liquid phase of reaction mixtures free from undissolved TPA |
| $w_{\text {EG }}^{\text {d }}$ | weight fraction of EG in the vapor phase |
| $w_{\mathrm{H}_{2} \mathrm{O}}^{d}$ | weight fraction of water in the vapor phase |
| $w_{\text {EG }}^{\prime}$ | weight fraction of EG in the reaction mixtures containing undissolved TPA; $w_{\text {EG }}^{\prime}$ $=\beta \cdot \omega_{\mathrm{EG}}$ |
| $w_{\mathrm{H}_{2} \mathrm{O}}^{\prime}$ | weight fraction of water in the reaction mixtures containing undissolved TPA; $w_{\mathrm{H}_{2} \mathrm{O}}^{\prime}$ $=\beta \cdot w_{\mathrm{H}_{2} \mathrm{O}}$ |
| W | total weights of reaction mixtures in the reactor, kg |
| $W_{e}$ | weight of liquid phase in the reaction mixtures, kg |

## Greek Symbols

$\beta \quad$ weight fraction of liquid phase in the reaction mixtures; $\beta=W_{e} /\left(W_{e}+T_{S}\right)$
$\delta_{\text {EG }}$ flow rate of EG added anew into the reactor per $1 \mathrm{~kg} / \mathrm{h}$ throughput based on PET polymer, ( $\mathrm{kg} / \mathrm{h}$ ) $/(\mathrm{kg} / \mathrm{h})$ (PET)
$\delta_{\mathrm{H}_{2} \mathrm{O}} \quad$ flow rate of water added anew into the $J$ th reactor per $1 \mathrm{~kg} / \mathrm{h}$ throughput based on PET polymer, (kg/h)/(kg/h)(PET)
$\eta_{\mathrm{EG}}$ equivalent concentration of $\mathrm{EG}, \eta_{\mathrm{EG}}=16.111 \mathrm{~mol} / \mathrm{kg}(\mathrm{EG})$
$\eta_{\mathrm{H}_{2} \mathrm{O}}$ equivalent concentration of water, $\eta_{\mathrm{H}_{2} \mathrm{O}}=55.494 \mathrm{~mol} / \mathrm{kg}\left(\mathrm{H}_{2} \mathrm{O}\right)$
$\theta$ reaction time, $h$
$\tau \quad$ mean residence time defined by $W / F_{S}$, h
$\phi \quad$ ratio of hydroxyl end groups to total end groups of dried oligomer; $\phi=O H V /(A V+O H V)$

## Subscripts

EG ethylene glycol
$\mathrm{H}_{2} \mathrm{O}$ water
TPA terephthalic acid

## Superscripts

d distillate (vapor phase)

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[^0]:    Considering that the reaction rate constants of $k_{7}, k_{8}$, and $k_{9}$ correspond to the encounter probability of each hydroxyl end group, the probability where one end group reacts with the others is proportional to the number of combinations for each end group given below.

